2.3 Polynomial and Synthetic Division

Long Division

ex1] $6x^3 - 19x^2 + 16x - 4$ divided by x-2

$$
\begin{array}{r|l}
 & 6x^2 - 7x + 2 \\
x-2 & 6x^3 - 19x^2 + 16x - 4 \\
 & -6x^3 + 12x^2 \\
 & \quad -7x^2 + 16x \\
 & \quad +7x^2 \mp 14x \\
 & \qquad 2x - 4 \\
 & \qquad -2x + 4 \\
 & \qquad\quad 0
\end{array}
$$

Solution
$6x^2 - 7x + 7$

ex2] x^3-1 divided by x-1

$$
\begin{array}{r|l}
 & x^2 + x + 1 \\
x-1 & x^3 + 0x^2 + 0x - 1 \\
 & -x^3 + x^2 \\
 & \quad x^2 + 0x \\
 & \quad -x^2 + x \\
 & \qquad x - 1 \\
 & \qquad -x + 1 \\
 & \qquad\quad 0
\end{array}
$$

Solution
$x^2 + x + 1$

ex3] $2x^4 + 4x^3 - 5x^2 + 3x - 2$ divided by $x^2 + 2x - 3$

$$
\begin{array}{r}
2x^2 - 2 \text{ R } 7x-8 \\
x^2+2x-3 \;\big)\; 2x^4+4x^3-5x^2+3x-2 \\
-2x^4 \mp 4x^3 + 3x^2 \\
\hline
-2x^2+3x-2 \\
+2x^2+4x \mp 6 \\
\hline
7x-8
\end{array}
$$

Synthetic division this is a nice short cut to long division

ax^3+bx^2+cx+d divided by $x-k$

$$\begin{array}{c|cccc} k & a & b & c & d \\ & & ak & 0 & 0 \\ \hline & a & 0 & 0 & r \end{array} \text{ - Remainder}$$

vertical pattern: add terms
Diagonal patter: multiply terms

Use synthetic division to divide the following:

EX 4] x^4-10x^2-2x+4 by $x+3$

$$\begin{array}{c|ccccc} -3 & 1 & 0 & -10 & -2 & 4 \\ & & -3 & 9 & 3 & -3 \\ \hline & 1 & -3 & -1 & 1 & 1 \end{array} \text{ -Remainder}$$

$$\boxed{x^3-3x^2-x+1+\frac{1}{x+3}}$$

EX 5] x^3+1 by $x+1$

$$\begin{array}{c|cccc} -1 & 1 & 0 & 0 & 1 \\ & & -1 & 1 & -1 \\ \hline & 1 & -1 & 1 & 0 \end{array}$$

$$\boxed{x^2-x+1}$$

Remainder Theorem

If a polynomial f(x) is divided by x-k, then the reminder is r=f(k)

you will get a point on your graph and can check by substitution

use the remainder theorem to evaluate the following function at x+2 x = -2

$f(x) = 3x^3 + 8x^2+5x - 7$ remainder is -9

$$
\begin{array}{r|rrrr}
-2 & 3 & 8 & 5 & -7 \\
 & & -6 & -4 & -2 \\
\hline
 & 3 & 2 & 1 & -9
\end{array}
$$

(-2, -9)

this is a point on the graph

Check $3(-2)^3+8(-2)^2+5(-2)-7$

$3(-8)+8(4)-10-7$

$-24+32-17$

$8-17$

-9 ✓

Factor theorem

a polynomial f(x) has a factor (x-k) if and only if f(k)=0

EX] show that (x-2) and (x+3) are factor $f(x) = 2x^4+7x^3-4x^2-27x-18$

Your remainder must be zero for this to be true!

$$
\begin{array}{r|rrrrr}
2 & 2 & 7 & -4 & -27 & -18 \\
 & & 4 & 22 & 36 & 18 \\
\hline
 & 2 & 11 & 18 & 9 & 0
\end{array}
$$

Remember

$x-2=0$

$x=2$

use in synthetic division

Now use 4 and synthetically divide again!

$$
\begin{array}{r|rrrr}
-3 & 2 & 11 & 18 & 9\ 0 \\
 & & -6 & -15 & -9\ 0 \\
\hline
 & 2 & 5 & 3 & 0\ 0
\end{array}
$$

$x+3=0$

$x=-3$

$2x^2+5x+3=0$

$(2x+3)(x+1)=0$

Other Zeros

$x=-\frac{3}{2}$ & -1